

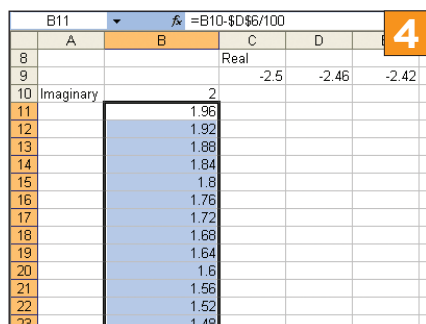
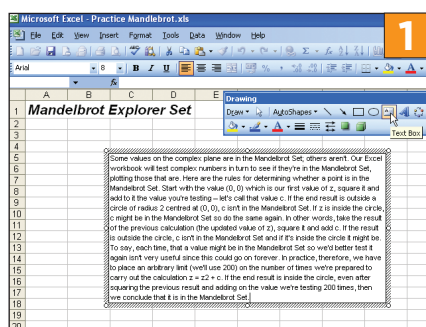
HOW TO... Explore the Mandelbrot Set

Most people only ever use a spreadsheet for their finances, but it can be used creatively for more artistic purposes, too. Here **Mike Bedford** shows how to use Excel to delve into the fascinating world of fractals

Excel isn't a program you'd tend to associate with creating art, nor is algebra something you'd normally associate with artistic endeavours. But maths can be used to create fractals, bizarre shapes that retain a lot of detail no matter how much you zoom in. Fractals can produce fascinating abstract images.

The Mandelbrot Set and other fractal figures can be created by programs, and there are plenty available on the market. As fascinating as these are, though, there's nothing like creating your own fractal from scratch, especially if you want to get a greater understanding of them. Here we'll show you how to draw the Mandelbrot Set and zoom right into it using Excel. We have used Excel 2003, but this walkthrough will work with earlier versions, too. To familiarise yourself with how the Mandelbrot Set works before you start, see the box opposite.

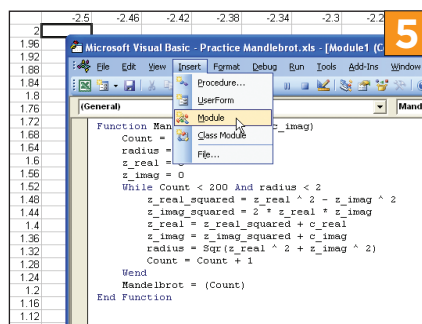
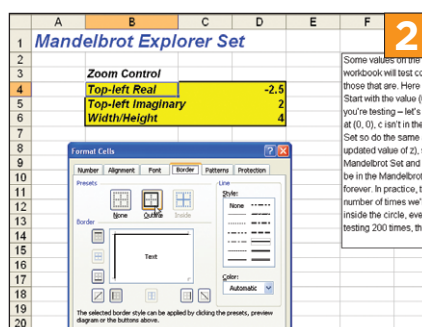
1 Start Excel and make sure you have a blank workbook onscreen. Type Mandelbrot Explorer Set into cell A1. Format the title with a font size of 16, also selecting bold and italics. In the Drawing toolbar (enable from View, Toolbars if it's not visible), click on the text box icon. Drag a box around a spare part of the workbook; you can always move it later. Enter a short description of the purpose of the workbook and how it works.



2 We'll automate zooming by providing boxes into which you can enter values for the top-left hand corner and the width and height of the portion of the complex plane you want to view. Enter the text 'Zoom Control' in cell B3, Top-left Real in cell B4, Top-left Imaginary in cell B5 and Width/Height in cell B6. We'll enter the initial values that let us see the entire Mandelbrot Set. Type -2.5, 2, and 4 into cells D4, D5 and D6 respectively. Format the text and cells to make them stand out.

3 Each time we plot the Mandelbrot Set, we'll do so with a grid of 101x101 dots. What we need, therefore, is a block of 101x101 cells bounded by the values defined by the numbers we entered in Step 2. Type Real into cell C8 and Imaginary into cell A10. Then type =D4 into cell C9 and =D5 into cell B10. The values -2.5 and 2 – the values for the real and imaginary components of the top-left-hand corner – appear in these cells.

4 Type =C9+\$D\$6/100 into cell D9. Click in that cell and, while holding down the mouse button, drag right until the cells D9 through to CY9 are highlighted. Release the mouse button. From the Edit menu, select Fill and Right. You'll see the cells D9 to CY9 now contain the values from -2.5 to 1.5 (that is, a

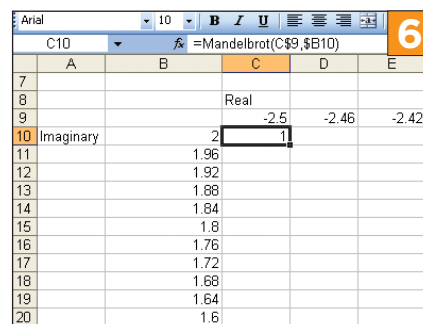
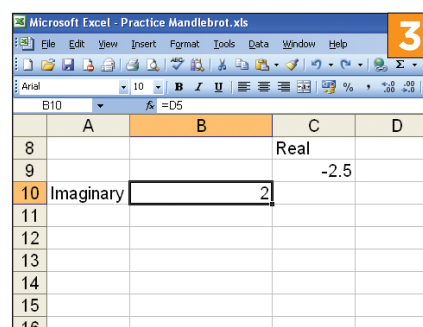


width of 4) in 100 steps. Similarly, type =B10-\$D\$6/100 into cell B11 and copy this formula down through all cells to B110.

5 To determine if a point is in the Mandelbrot Set, we need to carry out an operation repeatedly, ending when certain conditions are met. This isn't easy in Excel, but it is in a conventional programming language such as Visual Basic. You can define your own functions in Excel using Visual Basic, and that's our next job.

The function we're going to create is called Mandelbrot. Instead of just giving a result indicating whether or not a point is in the Mandelbrot Set, it will provide as its result the number of times the calculation $z = z^2 + c$ was carried out. A value of 200 indicates a point inside the Mandelbrot Set; other values indicate how close a point is to the Set. From the Tools menu, select Macro and then Visual Basic Editor. The Visual Basic Editor window appears. From the Insert menu, select Module. Type the following into the new window that appears; when finished, select Close and Return to Microsoft Excel from the File menu:

```
Function Mandelbrot(c_real, c_imag)
    count = 0
    radius = 0
```



```

z_real = 0
z_imag = 0
While count < 200 And radius < 2
  z_real_squared = z_real ^ 2 -
  z_imag ^ 2
  z_imag_squared = 2 * z_real *
  z_imag
  z_real = z_real_squared + c_real
  z_imag = z_imag_squared + c_imag
  radius = Sqr(z_real ^ 2 + z_imag
  ^ 2)
  count = count + 1
Wend
Mandelbrot = (count)
End Function

```

6 Enter =Mandelbrot(C\$9,B\$10) into cell C10. The value 1 appears in the cell. This is the result of giving our Function Mandelbrot the values of -2.5 and 2 as the real and imaginary parts of the top-left-hand corner of the square we defined in Step 2. That value of 1 indicates the point is well outside the Mandelbrot Set.

7 Click in cell C10 and, while holding down the mouse button, drag right until the cells C10 through to CY10 are highlighted. Release the mouse button. From the Edit menu, select Fill and then Right. Each cell contains the value of 1, indicating that all points on the top row of the square are well outside the Mandelbrot Set.

8 Highlight from cell C10 down to cell CY110. From the Edit menu, select Fill, Down. Within the next few seconds, all these cells will fill with a value indicating how close each point is to the Mandelbrot Set.

9 Click into cell B9 and, while holding down the mouse button, drag right and down until all the cells from B9 through to CY110 are highlighted. Click on the Chart icon to start the Chart Wizard. In Step 1 of the Chart Wizard, select Surface as the Chart type and the bottom-left entry as the Chart subtype before clicking Finish. A chart will appear showing the familiar

Fractal rules The Mandelbrot Set

The Mandelbrot is drawn on the complex plane, which plots all possible complex numbers against two axes. For an explanation of complex numbers, visit http://en.wikipedia.org/wiki/complex_numbers. Some values on the complex plane are in the Mandelbrot Set, while others aren't. Our Excel workbook tests complex numbers in turn to see if they're in the Mandelbrot Set, plotting those that are.

Here are the rules for determining whether a point is in the Mandelbrot Set. Start with the value (0, 0), our first value for z, then square it and add to it the value you're testing – let's call that value c. If the end result, our new value for z, is outside a circle of radius 2 centred at (0, 0), c isn't in the Mandelbrot Set. If z is inside the circle, c might be in the Mandelbrot Set, so do the same calculation again. In other words, take the updated value of z, square it and add c.

If the result is outside the circle, c isn't in the Mandelbrot Set, and if it's inside the circle it might be.

To say each time that a value might be in the Mandelbrot Set and we'd better test it again isn't useful, since this could go on forever. So we have to place an arbitrary limit (we'll use 200) on the number of times we're prepared to carry out the calculation $z = z^2 + c$. If the end result is inside the circle, even after squaring the previous result and adding on the value we're testing 200 times, we conclude that it is in the Mandelbrot Set.

With this set of rules you might expect the Mandelbrot Set to be a boring shape, perhaps a circle, but this couldn't be further from the truth. It's a strange shape, and as you look at its perimeter in greater detail (by zooming in on it) you find more and more detail. You could go on forever and with each zoom find more intricate detail.

shape of the Mandelbrot Set. This will be upside down, so right-click on the axis at the right-hand side and select Format Axis from the drop-down menu. Select the Scale tab in the Format Axis dialog box and click Series in reverse order, OK.

10 The chart shows colours for only four ranges of values. Right-click in the legend that appears to the right of the chart and select Format legend from the drop-down menu. Click on the Scale tab and enter 10 as the Major unit and 201 as the Maximum. The Mandelbrot Set is redrawn with more coloured contours around it.

11 Each coloured contour has a black border that detracts from the pattern, so we'll remove them. Resize the chart, so all the legend can be displayed. Select the legend by clicking on it, then double-click on the coloured box at the bottom. The Format Legend dialog box appears. On the Patterns tab, under Border, click None

then OK. Repeat for the other coloured boxes in the legend. We don't need the legend now, so select it and press the Del key.

12 If you look at the Mandelbrot Set, it's clear there's plenty of detail in a box 0.3 units wide with its top-left-hand corner at (-0.3, 1.2). So type in the values -0.3, 1.2 and 0.3 into cells D4, D5 and D6 and watch as this section of the Mandelbrot Set is drawn in greater detail. You can continue zooming in as much as you want. As long as you don't pick an area well inside or outside the Mandelbrot Set, you'll see more and more detail. If you're having difficulty understanding the Visual Basic code in Step 5, remember a complex number is of the form $x+yi$ where i is the square root of -1, x is the real part and y the imaginary part. Squaring the complex number $x+yi$ gives x^2-y^2+2xyi , which means the real part of the result is x^2-y^2 and the imaginary part is $2xy$. **E**

